COMPUTATIONAL EXPERIMENT TO SOLVE PROBLEMS OF OPTIMIZATION IN PULSED HEATING REGIMES

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We present the results from numerical solutions of problems in the optimum control of heat conduction in the case of pulsed heating with highly concentrated sources of heat.

INTRODUCTION

The problems of optimum control (POC) involved two mathematical models (MM): the MM of the direct problem (DC) and the MM of the POC itself. The MM of the DC is needed, since without it we cannot formulate the POC, nor can we examine those terms in the MM of the DC which make it possible to establish the limitations and controls. When we have at hand the MM of the DC for heat conduction (H), it is easy to formulate the POCH, since the latter is easily reduced to a "standard" optimization problem [1-4]. The MM of the DC can be written in implicit (MM of the Ist kind) and in explicit (MM of the IInd kind) form [5]. Selection of the method for the solution of the POCH depends significantly on the form of the MM for the DC. Indirect methods for the solution of POC can be applied easily, when we have obtained the explicit forms of the MM for the DC, i.e., analytic relationships between temperature and control (the conditions of unique definition). However, for complex implicit MM, even in the case of ordinary heat conduction, it is not so simple to come up with an analytical solution, and the numerical solutions are therefore processed in the form of nomograms or in the form of explicit analytical expressions. We then use any optimization algorithms of which in [1] alone we find about 300. We indicated our preference for the methods of zeroth order, since we were interested in the absolute temperatures and their discrepancies. Solutions of the DC and POC were obtained by means of all existing types of calculators and computers (CC): analog, universal, digital, and hybrid.

Physical and Mathematical Formulation of the Problem. The basic equations for the process of heat conduction and the expressions for boundary conditions of the Ist-IVth kind can be found in numerous monographs and texts dealing with heat exchange. We would stress that all of the quantities to which the conditions of uniqueness in the MM of the DCH apply are controls within the MM of the POCH. Specific controls and limitations are easily determined after specification of the conditions of uniqueness and selection of the form of the designated special-purpose function.

Figure 1 shows two canonical shapes: a disk and a plate. The areas subjected to pulsed heating are indicated by cross-hatching. The POC was solved on the basis of numerical solutions (explicit MM). In formal terms, all of the controls of our POC serve as arguments in the conditions of unique definition for the original DC.

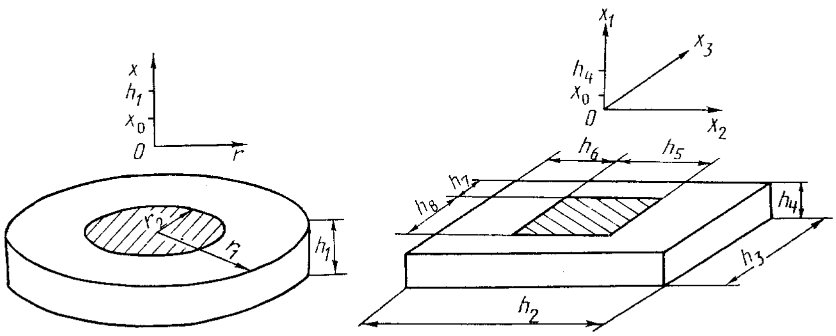

Fig. 1. Structural diagrams of the test specimens.

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Items in the shape of disks or plates are heated by pulsed surface or internal flows of heat. The laws governing the change in the power of the surface $q_s(x_1, \tau)$ (i = 1, 2) or internal $q_V(x_i, \tau)$ (i = 1, 2, 3) heat flows were set so as to make provision for the physical nature of the heating source. Over time the power is transmitted in the form of rectangular pulses exhibiting such characteristics as q_s, q_V, τ_1, τ_2, the number of pulses, the number of pulse trains, etc. In addition, the following quantities contained within the uniqueness conditions also served as controls, namely: x_i, i = 1, 2, 3, the shape of the body for any coordinate system in the cases which we have presented here varied r_1, r_2, h_1, h_2, h_3, h_4 (see Fig. 1). For multilayered anisotropic shapes we were able to change the number, thickness, and thermophysical characteristics of the individual layers. The purpose of the POCH series was to find such optimum shapes and thermophysical characteristics of the shapes where the maximum temperature T_{max} would be lower than the temperature specified throughout the duration of the entire process. One class of such POCH may be referred to as a geometric POC class. In these POCH we found controls with which reduction became possible, i.e., transition from three- and two-dimensional problems to one-dimensional problems. The problem is formulated as follows: with which controls can the temperature field obtained in the solution of the three- or two-dimensional problem be equivalent (for any specified discrepancy) to the field of the one-dimensional problem? In these problems we have revealed features which go under the heading of the "action" of the principle of local effect, i.e., the principle of relaxation in space and time for local changes in controls (conditions of uniqueness in DCH).

Thermal resistances to heat conduction $R_\lambda = \ell/(\lambda S)$, to heat capacity $R_\tau = \delta\tau/(c_V V)$, to external and internal exchanges of heat and to sources $R_\alpha = 1/(\alpha S)$, $R_{q_s} = (U_{max} - U_i)K/(q_s S)$, $R_{qV} = (U_{max} - U_i)K/(q_V V)$, to convection $R_u = \Delta x/(u c_V V)$, these may all serve as controls. The numerical values of these parameters and the values of their relationships are criteria for estimates of the thermal regimes, the thermal patterns for the test objects (and for heat exchangers in general). Some of these, for example $R_\lambda/R_\alpha = Bi = \alpha/(\lambda\ell)$, have long since been employed successfully in thermal engineering. For POCH these criteria are control ratios.

We have to solve the POCH and find such ratios of thermal resistances as the controls which alter the class of problems and the subsystem MM of the DC. We can formulate the POCH as follows: find such a value for the criterion as would allow us to "deform" the item: to make the changeover from a cylindrical round shell to a plate. Such a criterion is known: $h_1/r_1 < 0.1$, but in the POCH we have minimization $T_{max\,p} - T_{max\,a} \le \varepsilon$, i.e., the value of ε is specified and we solve the problem of minimizing the difference $T_{max\,p} - T_{max\,a}$. The ratio h_1/r_1 serves as the control. Thus, not only uniqueness conditions can serve as controls, but their combinations and ratios. In our case, these ratios are used to subject the items to "deformation," i.e., to simplifications of the thermal pattern and to simplifications of the MM DCH hierarchy that corresponds to this pattern.

Let us list some of the controls and limitations which we have investigated. Fundamentally, the aims and limitations in the POC are limitations, although the purposeful aims are usually isolated into an individual group of conditions.

The listed controls and limitations are as follows: $T_{max\,a} = 1000\text{-}1700°C$, ΔT_{Sa} is the allowable difference between the temperature at a given point x at a given instant of time τ and the integral temperature at which the deformation k of the product (Fig. 1) does not exceed some given quantity (k = 0.5-2 μm), $h_{1,2}$ = 4-30 mm, h_3 = 20-40, h_4 = 20-30; $h_{5,6}$ = 10, h_7 = 5-10, h_8 = 5-15 mm; r_2 = 5.6 mm, r_1 = 10-12.5 mm; $q_V = (0.5\text{-}30)\cdot 10^6$ W/m²; $\tau_1 = 5\cdot 10^{-5}\text{-}3.0$ sec, $\tau_2 = 5\cdot 10^{-3}\text{-}10$ sec. The indicated numerals represent the range of numerical values for the controls and correspond to the periods $P = 5\cdot 10^{-3}\text{-}11$ sec, frequencies ν = 0.1-200 Hz, and the number of pulses from one to continuous operation.

In certain regimes it is the times of the pulse trains and the separations between these trains that serve as controls. The number of pulses in the trains reached as high as 1000. The pores between the pulses and between the trains served as a control of time in the cooling down by means of radiation, and with natural and forced convection.

The difference between the maximum temperature T_{max} and the specified dangerous temperature T_{dan} was most frequently minimized, and here the controls had to be those quantities which were characteristics of the cooling (i.e., q_{cool} or α_{cool}, or T_a, etc.).

In principle, since the temperature depends on all of the unique conditions, all of these unique conditions are controls on the cooling. For example, a change in the conditions of heating q_V or τ_1, or τ_2, etc., are controls on cooling just as q_{cool}. The design estimator in the computational experiment may suggest to the designer all means of cooling, and the designer will select the most efficient, proceeding from the control of other POC (minimum of mass, expenditure of energy, cost, etc.).

In a number of cases the moving control is quite promising [6]. For all intents and purposes, we paid no attention to this control, with the exception of several variants of random displacement of the local heating spot with respect to

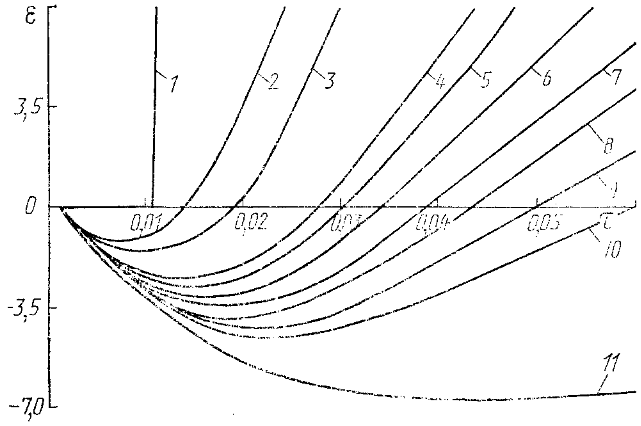

Fig. 2. Change over time in the discrepancy ε at the point $(h_1 - x) = 0.2$ mm for various q = const: 1) $q \cdot 10^{-6} = 4.474$; 2) 3.35; 3) 2.75; 4) 2.45; 5) 2.38; 6) 2.3; 7) 2.23; 8) 2.18; 9) 2.13; 10) 2.075; 11) 1.9 W/m^2. ε, °C; τ, sec.

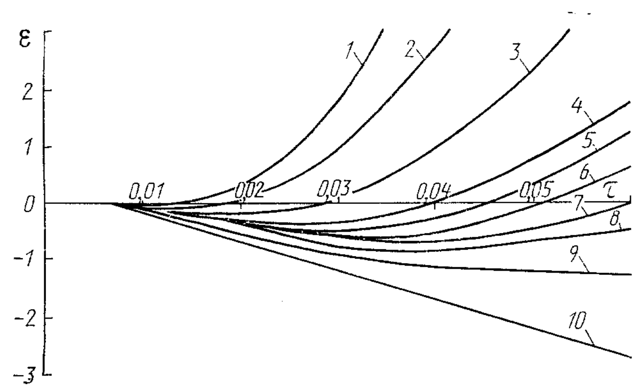

Fig. 3

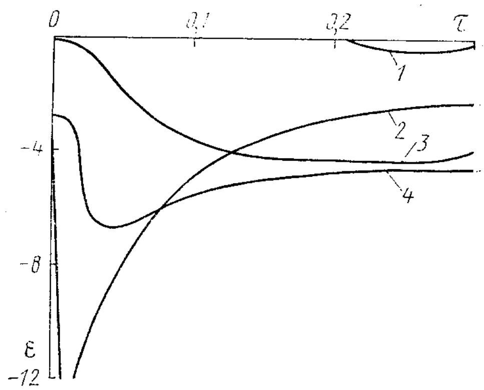

Fig. 4

Fig. 3. Discrepancy ε as a function of time at point $(h_1 - x) = 0.3$ mm for various q = const: 1) $q \cdot 10^{-6} = 4.0$; 2) 3.35; 3) 2.75; 4) 2.45; 5) 2.38; 6) 2.3; 7) 2.23; 8) 2.18; 9) 2.13; 10) 1.9 W/m^2.

Fig. 4. Maximum discrepancy $\varepsilon = T_c - T_v$ as a function of the time at points along the axis of the specimen (T_c has been obtained for $q = q_{e\ av} = 1.9 \cdot 10^6$; T_v has been obtained for $q_v = 19 \cdot 10^6$ W/m^2, $\tau_1 = 10^{-3}$, P = 0.1 sec): 1) $(h_1 - x) = 1.0$; 2) 0.1; 3) 0.3; 4) 0.2 mm.

the entire (possible) spots. Even these special examples of movable heating–cooling suggest the great potentials of such a control.

The initial MM of the DCH was nonlinear, i.e., we took into consideration the nonlinearity of the I and II kinds [$\lambda(T)$, $c_V(T)$, $q_{cool}(T)$]. For example, the emissivity ξ as a function of T could change by a factor of 4, from 0.2 to 0.8. By controlling the roughness of the heated surface, we can alter ξ and optimize the temperature fields.

The thermophysical characteristics of the materials were altered within the following limits: $\lambda = 1.1$–395 W/(m·K), $c_V = (2.54$–$4.2) \cdot 10^6$ J/(m^3·K). The characteristics λ and c_V in their dependence on temperature within the temperature ranges being studied here varied, respectively, by factors of 3 and 1.5. The computational experiment showed that errors due to linearization with respect to λ and c_V lead to an error in $T(x_i, \tau)$, in our DC reaching up to 30%. Simultaneously, in order to underscore the need to solve nonlinear problems and combined thermomechanical problems for our product, let us note that Young's modulus E and other characteristics of the mechanical state depend on temperature; it is essential that these relationships be taken into consideration for problems similar to ours [7].

It is impossible to achieve a solution for the POCH without an economical method for the solutions of the DCH, since the model temperatures T_M must be utilized in all methods for the solution of POC in the minimization of the special-purpose functional, where the values of T_m figure directly or indirectly (without limitation). Let us stress that the POCH are inverse problems in the most general sense of this word. Their solution is always associated with the inversion of the cause-and-effect coupling, that is incorrect from the physical point of view. From the mathematical standpoint, however, these inverse problems (ordinary OP and POC) are incorrect according to Adamar, and at times correct according to Tikhonov. In the POC for heat conduction, in view of the well-founded MM and the well-developed analytical and numerical methods for the solution of the DC the problems of control are solved with comparative simplicity, and this applies to problems of observability as well, etc. [8].

Results from the Solutions of Several POCH. Particular attention has been devoted to problems related to the search for equivalent constant flows of heat, which at a specific depth x_0 in the product would yield a temperature curve $T_c(x_0, \tau)$ (see Fig. 1) closest to the curve $T_v(x_0, \tau)$, obtained in the pulsed regime. Other formulations of the POC are important and also interesting from the standpoint of choosing optimum dimensions $r_{1(2)}$, h_i ($i = 1, ..., 8$). In these problems the ultimate goal was the attainment of minimum maximum temperature for given concentrated heating and distributed cooling heat flows. Such formulations would be of interest in connection with the fact that it would make it possible to ascertain specific thermal properties in the product, associated with system properties related to phenomena of heat treatment. The "effect of critical thickness" [9] has been confirmed: At some (critical) thickness the maximum temperature increases [rather than fall (!)] with a reduction in the thickness (h_1 or h_4).

The pulsed heat-exchange regimes are characteristic of numerous processes, and mathematical modeling of detailed temperature fields became an essential procedure, not only because of the great cost involved in carrying out experiments under natural conditions, but also because of the impossibility in such experiments completely to evaluate the thermal regime. The computational experiment becomes the only method for optimization. For example, if τ_1 is smaller by several orders of magnitude than the time required for the measuring unit to reach a normal operating regime, the change in maximum temperature cannot be measured by any existing methods.

In these and numerous similar cases we could pass judgment as to the operational capabilities of a design with respect to temperature, stress, strain, caused by a constant q rather than by pulses of heat flows, and the constant quantities result in thermal and thermomechanical regimes which are equivalent in terms of certain indices to the original pulsed regimes. For example, the thermal strains and stresses are evaluated on the basis of the temperature fields obtained at constant (but equivalent) q_{ce} [10]. The authors of [11] dealt with this principle of equivalence in the phenomenological theory of heat conduction with respect to making homogeneous the thermophysical properties. The method for the simplification of the MM of the DC serves as one of the means of optimizing the method for the solution of the initially complex problems of heat conduction in general, and the problem of pulsed heating in particular. The search for the constant q_{ce} yielding equivalent temperature fields on the basis of certain conditions is an optimum control problem in which the specified field is obtained for the pulsed regime, while the control of q_{ce} – const must result from our effort to find a minimum in the discrepancy in ε between two $T(x_0, \tau)$, obtained for the sought q_{ce} and q_v of the pulsed regime.

Let us examine several examples of solutions for such POC; we have enumerated the original data above. At some depth x_0 along the axis of the disk (Fig. 1) or at the inside (unheated) surface it is necessary to obtain for q_{ce} – const temperatures such as exhibit minimum difference from the temperatures obtained for q_v – varia. Figures 2-4 show the results from the solution of such problems. Along the axis of ordinates throughout we have plotted $\varepsilon = T_c - T_v$, where T_c is the temperature obtained for q – const; T_v has been obtained for q – varia, i.e., in pulsed heating. The coordinate of the point $(h_1 - x_0)$ changed from 1.0 to 0.1 mm. We can see that depending on the time and location of x_0 for identical $q_v = 19 \cdot 10^6$ W/m^2, $\tau_1 = 10^{-3}$, and $\tau_2 = 0.099$ sec we can find points for which $\varepsilon = 0$. The closer q_c to the value

$$q_{\text{e av}} = \sum_N \frac{q_v \tau_1}{\tau_1 + \tau_2},$$

the time-averaged q, the larger max ε over time (see curves 11 in Fig. 2). The data in the figures confirm the effect of the principle of local influence (the principle of relaxation of local perturbations in the conditions of unique deposition in space and in time). When the point x_0 is removed from the heated surface and with an increase in time the value of ε is reduced as the balance of the quantity of introduced heat is conserved. The growth in errors for q – const different from $q_{\text{e av}}$ (see Figs. 2 and 3) is characteristic. These curves once again give evidence as to the uniqueness of the solution for the heat-conduction problems and as to the possibility of obtaining q_i – const for the required instant of time for a given x_0. For example, we are interested in the point $(h_1 - x) = 0.2$ mm with an effective operational time of $\tau = 0.05$ sec for the pulse source. We can set q – const equal to $2.125 \cdot 10^6$ W/m^2, so that ε_{max} will be –4.5°C; here $\varepsilon = 0$ for τ – = 0.05 sec. We can take $\varepsilon_{max} = -3.5$°C, in which case $\varepsilon = 0$ for $\tau = 0.038$ sec, but when $\tau = 0.05$ sec it will be $\varepsilon = 3$ K. We can thus find such q – const that for a given depth of x in the specified range of times will yield the minimum possible ε_{max}.

For each pulsed regime q_v, τ_1, τ_2 we can find such a number of pulses at which q – const → $q_{\text{e av}}$. From the distribution of $T(x, \tau)$ we can approximately recover q_v, τ_1, τ_2, although the temperature curve, beginning from some x, fails to reflect the variability of q on the heating surface. It may be assumed that in this case the quantity q – const strongly reflects the pulsing nature of the heating regime and with some accuracy may serve as a solution of the inverse problem (essentially, we are dealing here with a problem of observability).

The curves in Figs. 2-4 provide a good picture of the laws of conservation: for all q – const larger than $q_{e\ av}$, the discrepancies in ε become positive even as $\tau \to \infty$, $\varepsilon \to \infty$.

The data in Fig. 4 are "perplexing": curve 2 for ε when $(h_1 - x) = 0.1$ mm behaves "incorrectly." From curves 4, 3, and 1 we can observe the following quantitative relationship: the larger $(h_1 - x)$, the smaller the value of ε. However, when $(h_1 - x) = 0.1$ with limited time $\varepsilon_{max}(h_1 - x) > \varepsilon_{max}$ for other $(h_1 - x)$, shown in Fig. 4. With $(h_1 - x) = 0.1$ mm the $\varepsilon(\tau)$ curves behave in the same "unusual" fashion because the curve $T(h_1 - 0.1, \tau)$ is considerably more "sensitive" to the pulse nature of the heating regime than the curves $T(x, \tau)$ for $(h_1 - x) = 0.2$, 0.3, and 1.0 mm. The data in Figs. 2-4 show that the temperatures through the depth of the heated bodies characterize the heating regime, and their values may be utilized to control heating.

CONCLUSION

Analysis of the POCH with respect to the reduction of two- and three-dimensional problems to one that is one-dimensional makes it possible to find such a number of pulses subsequent to which the reduction will lead to an impermissible growth in errors. We can demonstrate the number of pulses at which it becomes possible to achieve transition to calculations of one-dimensional temperature fields for three-dimensional bodies of complex shape. A complex shape is not only one that involves the shape of the body itself, but the shape of the cooling and heating surfaces as well. When we take into consideration the effect of the principle of relaxing local changes in the conditions of unique definition it becomes possible to optimize not only the thermal regime, but the method by which it is calculated, namely to simplify the original MM of the DCH, to reduce the calculation time for the specified high precision of solution for the DC and POC.

NOTATION

x, coordinate; q, heat flux density; T, T_a, $T_{c(v)}$, temperature, temperature of the medium, and the temperature obtained when q – const (q – varia); τ, τ_1, τ_2, time, pulse time, pause time; r_1, r_2, radii of component parts and the heating spot; h_i, geometric parameters of the body; R, thermal resistance; ℓ, length; S, area; V, volume; λ, thermal conductivity; c, specific heat capacity; $\delta\tau$, Δx, intervals of time and space; α, heat-transfer coefficient; U, electrical voltage; K, scale factor in calculation of thermal resistances; u, coolant velocity; Bi, Biot criterion; ε, discrepancy; ΔT_S, mean-integral deviation in temperatures; k, deformation; P, period; ν, frequency of pulse sequence; ξ, emissivity; N, number of pulses. Subscripts: s, surface; c, const; v, varia; p, purpose; a, allowable; dan, dangerous; cool, cooling; m, model; e, equivalent; min, minimum; av, average.

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